Sensitivity Analysis of Lossy Coupled Transmission Lines

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Abstract—**An analysis method, based on the numerical inversion of the Laplace transform, is described for the evaluation of the time domain sensitivity of networks which include lossy coupled transmission lines. The sensitivity can be calculated with respect to network components and parameters of the transmission lines. Sensitivity analysis is useful for waveform shaping and optimization. Examples and comparisons with sensitivity determined by perturbation are presented.**

I. Introduction

Analysis and design of interconnections in high-speed VLSI chips and printed circuit boards are gaining importance because of the rapid increase in operating frequencies and decrease in feature sizes. Improperly designed interconnects can result in increased signal delay, ringing, inadvertent and false switching [1]–[5]. This phenomenon can be observed at the chip as well as the system level and the interconnected blocks could be analog, digital or mixed. With higher signal speeds, electrical length of interconnects can become significant fraction of a wavelength. Consequently, the conventional lumped impedance interconnect model is not adequate in this case. Instead, a distributed transmission line model should be used.

A number of methods [6]–[9] have been proposed for the time domain analysis of networks which contain lossy coupled transmission lines. The common method found in the literature is based on separate equations formulated to describe the transmission line system and the terminal and interconnecting networks in the frequency domain. These equations are combined at the analysis stage. The time domain response is obtained by applying the inverse fast Fourier transform (FFT). The major difficulty with this approach is when the analysis has to span a time interval of several line transient times. For example, the response of a lossless line with short-circuited termination is of infinite duration. Consequently, it is impossible in this case to compute the response using FFT. Recently, an alternative method [10] for the analysis of lossy coupled transmission lines with arbitrary linear terminal and interconnecting networks has been proposed. This method is based on a unified formulation technique for the equations describing the transmission line system and the equations describing the terminal and interconnecting networks. The time domain response is obtained using numerical inversion of Laplace transform (NILT) [11], [12]. This method is more reliable than the FFT-based methods since it does not suffer from the usual aliasing problems.

The above methods are analysis or simulation tools which allow the circuit designer to determine the response of a given network. Human expertise is still required to study the effect of variance of network parameters on system performance. Critical components which can affect rise/fall times, overshoot/undershoot, crosstalk, delay, etc. have to be identified through repeated circuit simulations. An important step is to use optimization techniques to speed up the design process. Transmission line effects such as crosstalk, delay and reflection can be minimized at vital connection ports. Application of powerful gradient based optimizers depends on the knowledge of sensitivities of the output responses. The direct perturbation approach of sensitivity analysis reduces the efficiency of optimization and can be very expensive if the number of optimization variables is large.

In this paper, a new method to evaluate the sensitivity of networks containing lossy coupled transmission lines is presented. The proposed approach is based on the numerical inversion of Laplace transform and is more efficient and reliable compared to other methods based on FFT or the brute force perturbation technique. The developed tool can be used to identify critical network components or provide gradients to optimization routines.

In Section II, a unified approach based on the modified nodal admittance (MNA) matrix is used to formulate the equations describing the transmission line system and the equations describing the terminal and interconnecting networks. The frequency domain solution for the network and its adjoint are obtained at prespecified frequency points. These prespecified points are determined by the NILT algorithm which is described in Section III. In Section IV the frequency domain sensitivity with respect

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to transmission line parameters is evaluated in terms of the sensitivity of the eigenvalues and eigenvectors of the propagation matrix. Sensitivities to physical parameters of the transmission line are also taken into account. The time domain response of the network and its sensitivity are obtained from the frequency domain solution using numerical inversion of Laplace transform. In Section V, a summary of the computational steps is given. Finally, Section VI provides a number of examples comparing the sensitivity calculated using the proposed method and perturbation.

II. Formulation of the Network Equations

Consider a linear network π which contains linear lumped components and arbitrary linear subnetworks. The linear lumped components can be described by equations in either the time or frequency domain. The arbitrary linear subnetworks may contain frequency dependent or distributed elements that are best described in the frequency domain. Without loss of generality the modified nodal admittance matrix [13] equation of the network π can be written as

$$C_\pi \frac{dv_\pi(t)}{dt} + G_\pi v_\pi(t) + \sum_{k=1}^{N_s} D_k i_k(t) - e_\pi(t) = 0 \quad (1)$$

where

$C_\pi(t) \in \Re^{N_\pi \times N_\pi}, G_\pi(t) \in \Re^{N_\pi \times N_\pi}$ are constant matrices with entries determined by the lumped linear components,

$v_\pi(t) \in \Re^{N_\pi}$ is the vector of node voltage waveforms appended by independent voltage source current and inductor current waveforms,

$D_k = [d_{i,j}], d_{i,j} \in \{0, 1\}, i \in \{1, 2, \cdots, N_\pi\}, j \in \{1, 2, \cdots, n_k\}$ with a maximum of one nonzero in each row or column is a selector matrix that maps $i_k(t) \in \Re^{n_k}$, the vector of currents entering the linear subnetwork k, into the node space $\Re^{N_\pi}$ of the network π,

N_s is the number of linear subnetworks and

$e_\pi(t) \in \Re^{N_\pi}$ is the vector of source waveforms.

The frequency domain representation can be obtained by taking the Laplace transform of (1):

$$[sC_\pi + G_\pi]V_\pi(s) + \sum_{k=1}^{N_s} D_k I_k(s) = E_\pi(s) + C_\pi v_\pi(0). \quad (2)$$

Assume the frequency domain equations of the linear subnetwork k to be in the form

$$A_k V_k(s) = I_k(s) \quad (3)$$

where $V_k(s)$ and $I_k(s)$ represent the frequency domain terminal voltages and currents of the subnetwork k and A_k represents the MNA matrix of the subnetwork.

Combining (2) and (3) produces

$$Y_\pi V_\pi = E_\pi + C_\pi v_\pi(0) \quad (4)$$

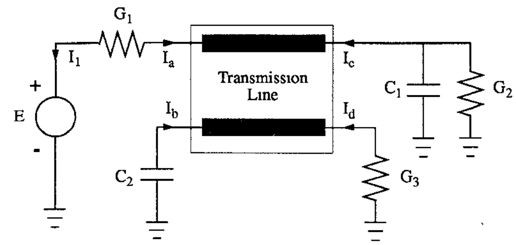

Fig. 1. Example circuit used to illustrate formulation.

where

$$Y_\pi = sC_\pi + G_\pi + \sum_{k=1}^{N_s} D_k A_k D_k^t.$$

Example: The following matrices illustrate the formulation for the network shown in Fig. 1:

$$G_\pi = \begin{bmatrix} G_1 & -G_1 & 0 & 0 & 0 & 1 \\ -G_1 & G_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & G_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & G_3 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C_\pi = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & C_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & C_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$V_\pi = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ I_1 \end{bmatrix} D_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} I_1 = \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_d \end{bmatrix} E_\pi = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ E \end{bmatrix}.$$

III. Time Domain Response Using Numerical Inversion of Laplace Transform

The application of the numerical inversion of Laplace transform to the time domain analysis of lossy coupled transmission lines is described in [10]. The method involves the computation of the frequency domain function at preassigned complex points and forming a weighted sum. It exactly inverts a certain number of terms of the Taylor series expansion of the time response and is thus equivalent to the methods used for the integration of differential equations. It has been shown that the method is absolutely stable and that the equivalent order of integration can be changed between 1 and 46 without affecting the stability properties. The time domain response for real time functions can be evaluated using the following equation:

$$\hat{v}(t) = -(1/t) \sum_{i=1}^{M'} \mathrm{Re}\left[K_i' V(z_i/t)\right]; t > 0 \quad (5)$$

where M', K_i' and z_i are related to the Padé approximation of the exponential function

$$e^z \approx \frac{P_N(z)}{Q_M(z)} = \sum_{i=1}^{M} \frac{K_i}{z - z_i} \tag{6}$$

and $P_N(z)$ and $Q_M(z)$ are polynomials of order N, M, respectively. If M is even $M' = M/2$ and $K_i' = 2K_i$ otherwise $M' = (M+1)/2$ and $K_i' = K_i$ for the residue corresponding to the real pole.

Given the Laplace domain MNA representation for a linear network containing lossy multiconductor transmission lines described in (4), the time domain response is

$$\hat{v}_\pi(t) = -(1/t) \sum_{i=1}^{M'} \text{Re}\{K_i' Y_\pi^{-1}(z_i/t) \cdot [E_\pi(z_i/t) + C_\pi v_\pi(0)]\}. \tag{7}$$

The response at each time point is obtained from the M' solutions to the network equation evaluated at the complex frequencies $s = z_i/t$. From (7) it can be seen that the solution at time t is completely independent of solutions at all other time points. If the circuit response for time t is all that is required this response can be calculated efficiently without calculating the response for any other values of time. Using inverse FFT techniques the entire waveform must be calculated.

IV. Sensitivity Analysis

The time domain sensitivity of a linear circuit requires the computation of the frequency domain solution at prespecified complex frequencies. The adjoint method for sensitivity analysis [14] provides an efficient method to calculate the frequency domain sensitivity of a linear network with respect to a parameter λ. Consider the system of linear equations described by (4). Define the output of the system to be

$$\phi = \boldsymbol{d}^t \boldsymbol{V}_\pi \tag{8}$$

where $\boldsymbol{d}$ is a constant vector.

Differentiating (4) and substituting (8) results in the final form of the sensitivity equation:

$$\frac{\partial \phi}{\partial \lambda} = (V_\pi^a)^t \left[\frac{\partial Y_\pi}{\partial \lambda} V_\pi - \frac{\partial E_\pi}{\partial \lambda} - \frac{\partial C_\pi}{\partial \lambda} v_\pi(0) \right] \tag{9}$$

where $Y_\pi^t V_\pi^a = -\boldsymbol{d}$. The solution to the adjoint vector V_π^a can be obtained with little additional cost because the LU factors of Y_π are already available from the solution of the linear network. Only forward/backward substitution is required to obtain V_π^a. The next step in determining the network sensitivity requires the calculation of $\partial Y_\pi/\partial \lambda$ and $\partial E_\pi/\partial \lambda$.

A. Sensitivity with Respect to Lumped Components

Network sensitivities to a specific component requires the evaluation of $\partial Y_\pi/\partial \lambda$. In the case of lumped components such as resistors, capacitors and inductors, the vector $\partial E_\pi/\partial \lambda$ is zero and

$$\frac{\partial Y_\pi}{\partial \lambda} = s\frac{\partial C_\pi}{\partial \lambda} + \frac{\partial G_\pi}{\partial \lambda}. \tag{10}$$

Example: Let λ be a conductance G_1 connected between nodes i and j. The MNA matrix will contain entries for G_1 as follows:

$$G_\pi = \begin{matrix} & i & j \\ i & G_1 & -G_1 \\ j & -G_1 & G_1 \end{matrix}. \tag{11}$$

Differentiating (11) with respect to G_1 and substituting into (10) yields

$$\frac{\partial Y_\pi}{\partial \lambda} = (e_i - e_j)(e_i - e_j)^t \tag{12}$$

where e_i is a vector containing the value one in location i and zero elsewhere.

If λ is a transmission line parameter the evaluation of $\partial Y_\pi/\partial \lambda$ is relatively more complex and will be described in the next section.

B. Sensitivity with Respect to Distributed Components

The lossy multiconductor transmission line [10] is assumed to be uniform along its length with an arbitrary cross section. The cross section of transmission line k with N_k signal conductors can be represented by the following $N_k \times N_k$ matrices of line parameters: the inductance per unit length $\boldsymbol{L}$, the resistance per unit length $\boldsymbol{R}$, the capacitance per unit length $\boldsymbol{C}$ and the conductance per unit length $\boldsymbol{G}$.

Let γ_i^2 be an eigenvalue of the matrix $Z_L Y_L$ with an associated eigenvector x_i, where

$$Z_L = R + sL \tag{13}$$

$$Y_L = G + sC. \tag{14}$$

It can be shown that the terminal voltages (3) are related by

$$A_k V_k = I_k \tag{15}$$

where

$$A_k = \begin{bmatrix} S_i E_1 S_v^{-1} & S_i E_2 S_v^{-1} \\ S_i E_2 S_v^{-1} & S_i E_1 S_v^{-1} \end{bmatrix}. \tag{16}$$

E_1 and E_2 are diagonal matrices

$$E_1 = \text{diag}\left\{ \frac{1 + e^{-2\gamma_i D}}{1 - e^{-2\gamma_i D}}, i = 1 \cdots N_k \right\} \tag{17}$$

$$E_2 = \text{diag}\left\{ \frac{2}{e^{-\gamma_i D} - e^{\gamma_i D}}, i = 1 \cdots N_k \right\}. \tag{18}$$

D is the length of the line,
S_v is a matrix with the eigenvectors x_i in the columns,
$S_i = Z_L^{-1} S_v \Gamma$ and
Γ is a diagonal matrix with $\Gamma_{i,i} = \gamma_i$.

Combining (4) and (9) produces

$$\frac{\partial \phi}{\partial \lambda} = (V_\pi^a)^t \left[D_k \frac{\partial A_k}{\partial \lambda} D_k^t \right] V_\pi = (V_k^a)^t \frac{\partial A_k}{\partial \lambda} V_k \tag{19}$$

where $V_k^a = D_k^t V_\pi^a$.

To find $\partial A_k / \partial \lambda$, rewrite (16) as

$$A_k \begin{bmatrix} S_v & 0 \\ 0 & S_v \end{bmatrix} = \begin{bmatrix} S_i & 0 \\ 0 & S_i \end{bmatrix} \begin{bmatrix} E_1 & E_2 \\ E_2 & E_1 \end{bmatrix}. \tag{20}$$

Differentiating (20) with respect to λ:

$$\frac{\partial A_k}{\partial \lambda} \begin{bmatrix} S_v & 0 \\ 0 & S_v \end{bmatrix} = \begin{bmatrix} \frac{\partial S_i}{\partial \lambda} & 0 \\ 0 & \frac{\partial S_i}{\partial \lambda} \end{bmatrix} \begin{bmatrix} E_1 & E_2 \\ E_2 & E_1 \end{bmatrix} + \begin{bmatrix} S_i & 0 \\ 0 & S_i \end{bmatrix} \begin{bmatrix} \frac{\partial E_1}{\partial \lambda} & \frac{\partial E_2}{\partial \lambda} \\ \frac{\partial E_2}{\partial \lambda} & \frac{\partial E_1}{\partial \lambda} \end{bmatrix} - A_k \begin{bmatrix} \frac{\partial S_v}{\partial \lambda} & 0 \\ 0 & \frac{\partial S_v}{\partial \lambda} \end{bmatrix} \tag{21}$$

where

$$Z_L \frac{\partial S_i}{\partial \lambda} = \frac{\partial S_v}{\partial \lambda} \Gamma + S_v \frac{\partial \Gamma}{\partial \lambda} - \frac{\partial Z_L}{\partial \lambda} S_i.$$

From (21), it can be seen that $\partial A_k / \partial \lambda$ is dependent on the sensitivity of the eigenvalues γ_i^2 and eigenvectors x_i of the matrix $Z_L Y_L$. Let

$$(\gamma_i^2 U - Z_L Y_L) x_i = 0 \tag{22}$$

where U is the identity matrix.

Differentiating (22) with respect to λ produces

$$\frac{\partial \gamma_i^2}{\partial \lambda} x_i + \gamma_i^2 \frac{\partial x_i}{\lambda} - \frac{\partial (Z_L Y_L)}{\partial \lambda} x_i - Z_L Y_L \frac{\partial x_i}{\partial \lambda} = 0. \tag{23}$$

This is a system of N_k equations with $N_k + 1$ unknowns. To solve the system one more equation has to be defined. An eigenvector can be normalized as follows:

$$x_i^t x_i = 1. \tag{24}$$

Differentiating (24):

$$x_i^t \frac{\partial x_i}{\partial \lambda} = 0. \tag{25}$$

Now with (25) inserted into (23), a solution for the partial derivatives of the eigenvalues and eigenvectors can be found.

$$\begin{bmatrix} \gamma_i^2 U - Z_L Y_L & x_i \\ x_i^t & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial x_i}{\partial \lambda} \\ \frac{\partial \gamma_i^2}{\partial \lambda} \end{bmatrix} = \begin{bmatrix} \frac{\partial (Z_L Y_L)}{\partial \lambda} x_i \\ 0 \end{bmatrix}. \tag{26}$$

To complete the solution, $\partial E_1 / \partial \lambda$ and $\partial E_2 / \partial \lambda$ have to be evaluated. Differentiating E_1 and E_2 produces

$$\frac{\partial E_1}{\partial \lambda} = \mathrm{diag}\left\{ \frac{-4\left(\frac{\partial \gamma_i}{\partial \lambda} D + \gamma_i \frac{\partial D}{\partial \lambda} \right) e^{-2\gamma_i D}}{(1 - e^{-2\gamma_i D})^2} \right\} \tag{27}$$

$$\frac{\partial E_2}{\partial \lambda} = \mathrm{diag}\left\{ \frac{2\left(\frac{\partial \gamma_i}{\partial \lambda} D + \frac{\partial D}{\partial \lambda} \gamma_i \right)(e^{-\gamma_i D} + e^{\gamma_i D})}{(e^{-\gamma_i D} - e^{\gamma_i D})^2} \right\}. \tag{28}$$

Once all the separate partial derivatives have been calculated and combined, the resultant $\partial A_k / \partial \lambda$ can be inserted into (19) to produce the frequency domain sensitivity $\partial \phi / \partial \lambda$. It should be noted that λ could be an electrical parameter of the transmission line or a physical parameter.

C. Sensitivity with Respect to Physical Parameters

Sensitivity with respect to the physical parameters of the transmission line can be determined by extending the algorithm described in the previous section. Physical parameters include length of the transmission line, width of the conductors, distance separating conductors, etc.

Determining the sensitivity to physical parameters requires the evaluation of the sensitivity of the electrical parameters of the transmission line to the physical parameter. The electrical parameters can be obtained numerically [15]–[17] or, in some special cases, by using explicit empirical formula [18]. In either case, using chain rule expansion, the sensitivity of the output can be expressed as

$$\frac{\partial \phi}{\partial \lambda} = \sum_{i=1}^{N_k} \sum_{j=1}^{N_k} \left[\frac{\partial \phi}{\partial R_{i,j}} \frac{\partial R_{i,j}}{\partial \lambda} + \frac{\partial \phi}{\partial L_{i,j}} \frac{\partial L_{i,j}}{\partial \lambda} + \frac{\partial \phi}{\partial G_{i,j}} \frac{\partial G_{i,j}}{\partial \lambda} + \frac{\partial \phi}{\partial C_{i,j}} \frac{\partial C_{i,j}}{\partial \lambda} \right]. \tag{29}$$

When determining sensitivity to the length D of transmission line k, the calculation of $\partial A_k / \partial D$ can be simplified. From (26), the sensitivity of the eigenvalues and

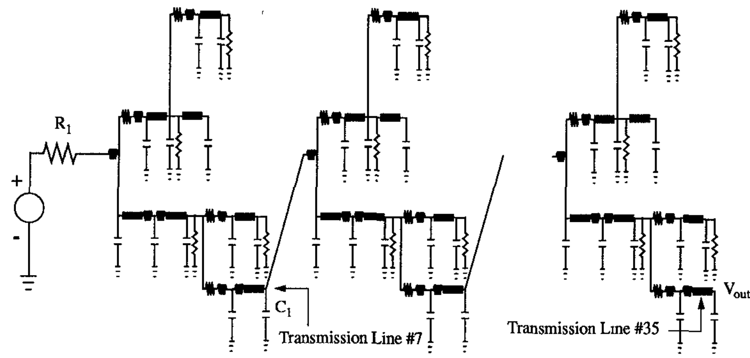

Fig. 2. Circuit for example 1, obtained by cascading circuit shown in Fig. 3.

eigenvectors to D is zero. Therefore

$$\frac{\partial \boldsymbol{A}_k}{\partial D} = \begin{bmatrix} \boldsymbol{S}_i \frac{\partial \boldsymbol{E}_1}{\partial D} \boldsymbol{S}_v^{-1} & \boldsymbol{S}_i \frac{\partial \boldsymbol{E}_2}{\partial D} \boldsymbol{S}_v^{-1} \\ \boldsymbol{S}_i \frac{\partial \boldsymbol{E}_2}{\partial D} \boldsymbol{S}_v^{-1} & \boldsymbol{S}_i \frac{\partial \boldsymbol{E}_1}{\partial D} \boldsymbol{S}_v^{-1} \end{bmatrix} \tag{30}$$

where

$$\frac{\partial \boldsymbol{E}_1}{\partial D} = \text{diag}\left\{ \frac{-4\gamma_i e^{-2\gamma_i D}}{(1-e^{-2\gamma_i D})^2} \right\} \tag{31}$$

$$\frac{\partial \boldsymbol{E}_2}{\partial D} = \text{diag}\left\{ \frac{2\gamma_i (e^{-\gamma_i D} + e^{\gamma_i D})}{(e^{-\gamma_i D} - e^{\gamma_i D})^2} \right\}. \tag{32}$$

D. Time Domain Sensitivity using NILT

Given a frequency domain response for the circuit sensitivity $\partial\phi(s)/\partial\lambda$ and using (5), the time domain response is

$$\frac{\partial \hat{\phi}(t)}{\partial \lambda} = -(1/t) \sum_{i=1}^{M'} \text{Re}\left[K_i' \frac{\partial \phi(z_i/t)}{\partial \lambda} \right]. \tag{33}$$

The response at each time point is obtained from the M' solutions to the sensitivity equation evaluated at the complex frequencies $s = z_i/t$.

V. Summary of Computational Steps

The following is a summary of the sensitivity analysis technique proposed in this paper:

Formulate $\boldsymbol{Y}_\pi$;
For $j = 1$ to number of time points
  For $i = 1$ to M'
    Solve for $\boldsymbol{V}_\pi$ and $\boldsymbol{V}_\pi^a$ at $s = z_i/t_j$;
    if λ is a transmission line parameter
      Calculate $\partial \boldsymbol{A}_k/\partial\lambda$ using (21);
      Calculate $\partial\phi/\partial\lambda$ using (19);
    else
      Calculate $\partial \boldsymbol{Y}_\pi/\partial\lambda$ using (10);
      Calculate $\partial\phi/\partial\lambda$ using (9);
    end;
    Add V_π and $\partial\phi/\partial\lambda$ to the weighted sums of (7) and (33);
  end;
end.

VI. Examples

In this section a number of examples are presented. To show the accuracy of the proposed method, each example has been verified by perturbing the parameter λ. The sensitivity plots have been normalized to represent mV change per 1 percent change in λ. It should be mentioned that the perturbation technique is computationally expensive since it requires an extra time domain analysis for each parameter λ, whereas using the proposed method the additional cost to evaluate the sensitivity with respect to *all* parameters is less than the computational cost of *one* time domain analysis.

Example 1: The circuit in Fig. 2 contains 35 single lossless transmission lines. The circuit is obtained by cascading the network in Fig. 3. The applied voltage is a pulse shown in Fig. 4 and the time response of the voltage V_{out} is shown in Fig. 5. The time domain sensitivities of the output voltage V_{out} evaluated using the proposed method are shown in Figs. 6–9 which demonstrate excellent agreement with the results obtained by perturbation using HSPICE. The CPU time comparison is listed in Table I. The comparison was made on a Sun SPARCstation IPC.

Example 2: The circuit shown in Fig. 10 contains two coupled lines highly interconnected by a terminal network. The first transmission line has four conductors, is 0.3 meters in length, and is described by the following parameters:

$$\boldsymbol{L} = \begin{bmatrix} 1.5 & 0.18 & 0 & 0 \\ 0.18 & 1.5 & 0.18 & 0 \\ 0 & 0.18 & 1.5 & 0.18 \\ 0 & 0 & 0.18 & 1.5 \end{bmatrix} \text{uH/m},$$

$$\boldsymbol{C} = \begin{bmatrix} 0.266 & -0.02 & 0 & 0 \\ -0.02 & 0.266 & -0.02 & 0 \\ 0 & -0.02 & 0.266 & -0.02 \\ 0 & 0 & -0.02 & 0.266 \end{bmatrix} \text{nF/m}$$

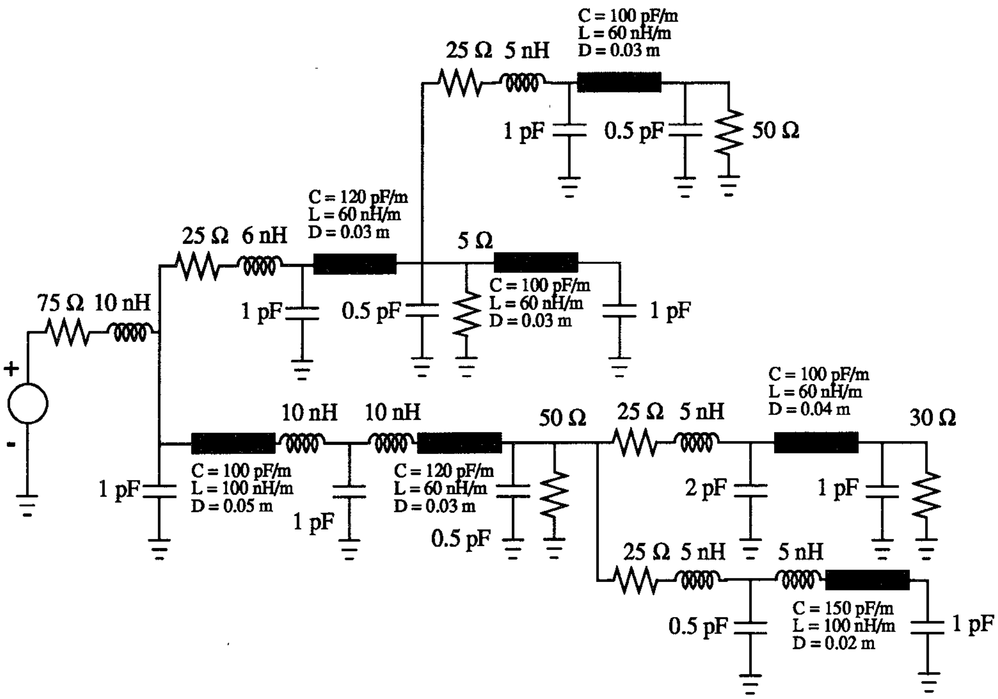

Fig. 3. Basic circuit block for example 1, transmission lines with terminal networks.

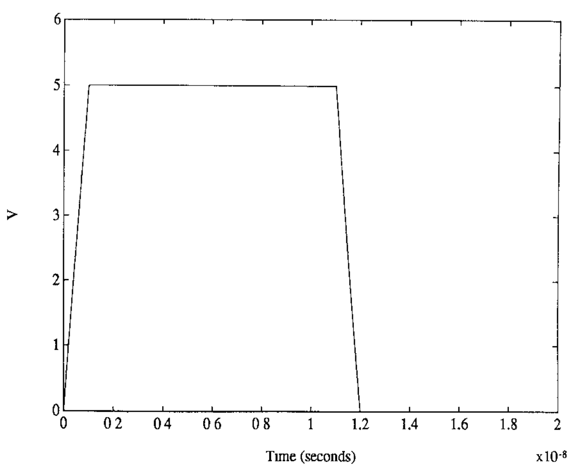

Fig. 4. Applied input voltage waveform used for example 1.

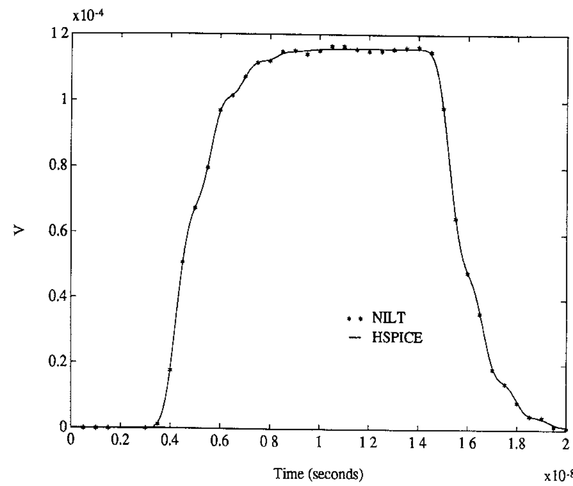

Fig. 5. Transient response of the circuit shown in Fig. 2 at node V_{out}.

and $\boldsymbol{R}=0$, $\boldsymbol{G}=0$. The second transmission line has two conductors, is 0.4 meters in length and is described by the following parameters:

$$\boldsymbol{L}=\begin{bmatrix} 750 & 95 \\ 95 & 750 \end{bmatrix} \text{nH/m},$$

$$\boldsymbol{C}=\begin{bmatrix} 0.133 & -0.009 \\ -0.009 & 0.133 \end{bmatrix} \text{nF/m}$$

and $\boldsymbol{R}=0$, $\boldsymbol{G}=0$.

Fig. 11 is a plot of the applied voltage. The circuit response of the voltage V_{out} using NILT is shown in Fig. 12. The time domain sensitivities of the output voltage V_{out} are compared with the perturbation method in Figs. 13–16.

Example 3: The circuit in Fig. 17 contains three lossy coupled transmission lines. The lengths of transmission lines 1, 2, and 3 are 0.05 m, 0.04 m and 0.03 m respectively. The transmission line parameters for the three lines are identical:

$$\boldsymbol{L}=\begin{bmatrix} 494.6 & 63.3 \\ 63.3 & 494.6 \end{bmatrix} \text{nH/m},$$

$$\boldsymbol{C}=\begin{bmatrix} 62.8 & -4.9 \\ -4.9 & 62.8 \end{bmatrix} \text{pF/m},$$

$$\boldsymbol{R}=\begin{bmatrix} 75 & 15 \\ 15 & 75 \end{bmatrix} \Omega/\text{m}, \qquad \boldsymbol{G}=\begin{bmatrix} 0.1 & -0.01 \\ -0.01 & 0.1 \end{bmatrix} \text{S/m}.$$

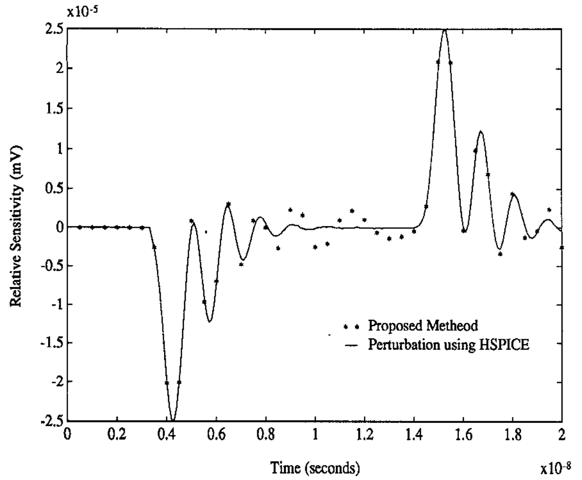

Fig. 6. Sensitivity of voltage V_{out} with respect to C_1 of Fig. 2.

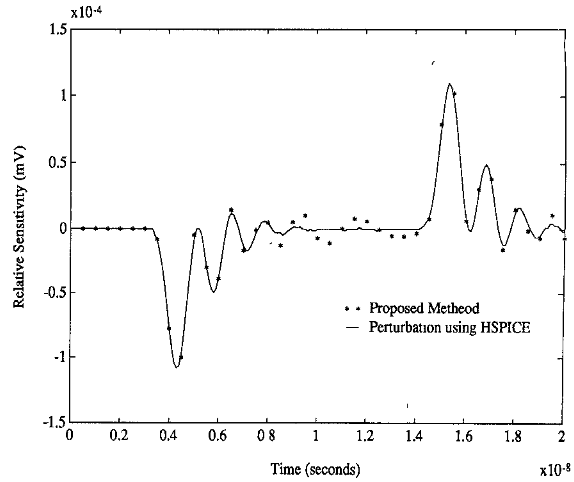

Fig. 9. Sensitivity of voltage V_{out} with respect to the capacitance per unit length of transmission line #35 in Fig. 2.

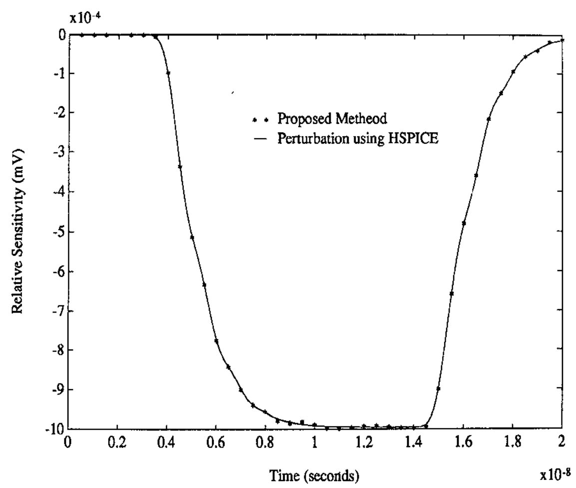

Fig. 7. Sensitivity of voltage V_{out} with respect to R_1 of Fig. 2.

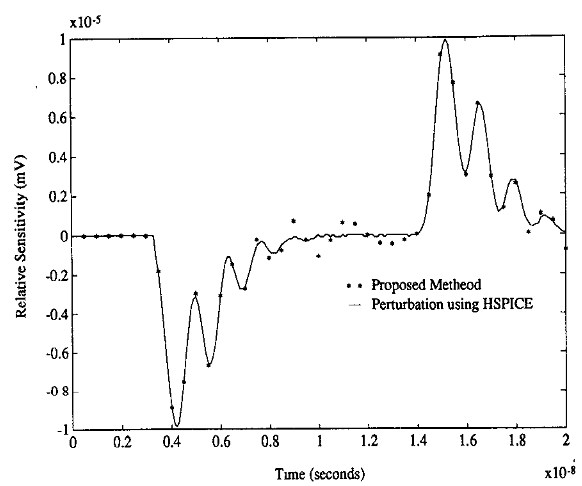

Fig. 8. Sensitivity of voltage V_{out} with respect to the inductance per unit length of transmission line #7 in Fig. 2.

TABLE I
CPU COMPARISON BETWEEN HSPICE AND PROPOSED METHOD

Number of Parameters	CPU Seconds		Speedup Ratio
	HSPICE	Proposed Method	
10	382.8	34.3	11.2
All	7516.8	60.3	124.7

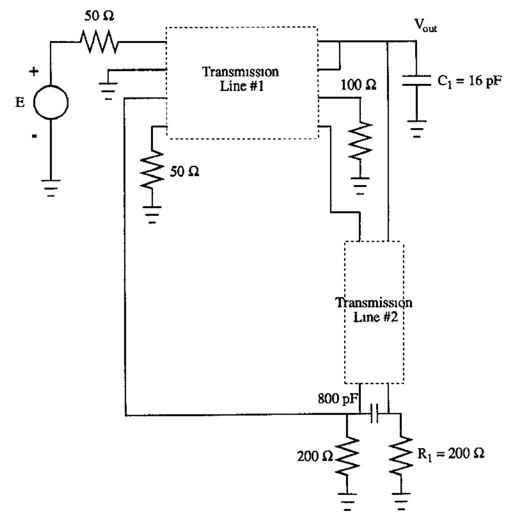

Fig. 10. Circuit for example 2, two multiconductor transmission lines with terminal networks.

The applied voltage for this example is shown in Fig. 11 and the circuit response is shown in Fig. 18. The time domain sensitivities of the output voltage V_{out} are compared with perturbation in Figs. 19–22.

Example 4: In [18] formulae for calculating the inductance and capacitance of the transmission line shown in Fig. 23 are provided. The transmission line consists of two flat conductors near a ground plane. Assuming a lossless transmission line, the transmission line parameters can be

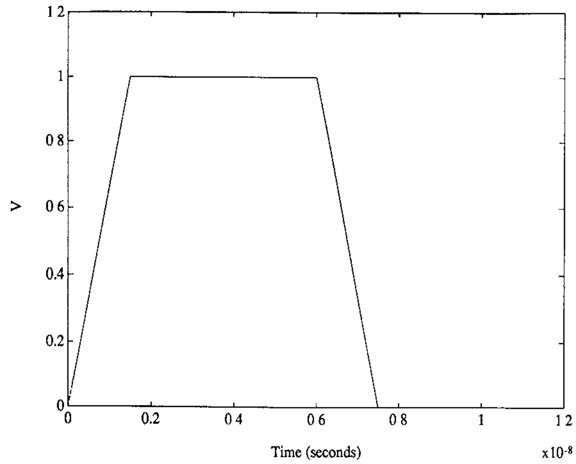

Fig. 11. Applied input voltage waveform used for examples 2 and 3.

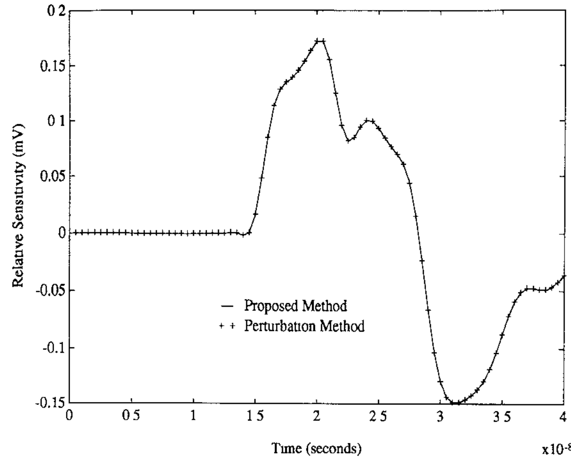

Fig. 14. Sensitivity of voltage V_{out} with respect to R_1 of Fig. 10.

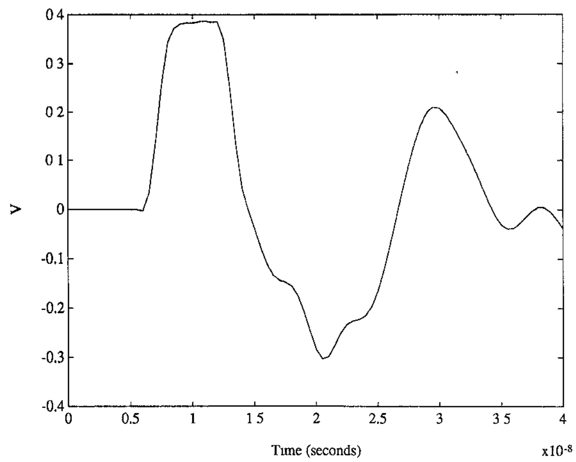

Fig. 12. Transient response of the circuit shown in Fig. 10 at node V_{out}.

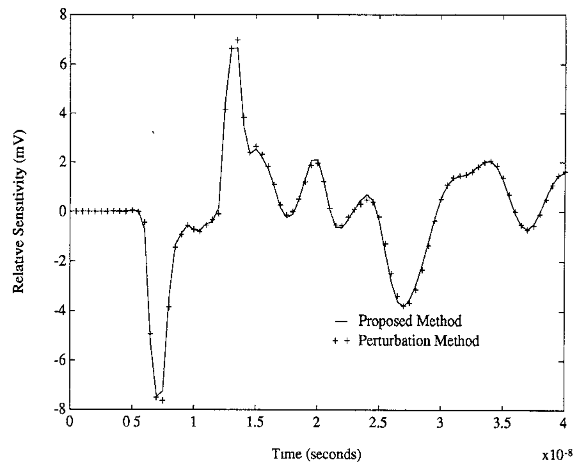

Fig. 15. Sensitivity of voltage V_{out} with respect to parameter $L_{1,1}$ of transmission line #1 in Fig. 10.

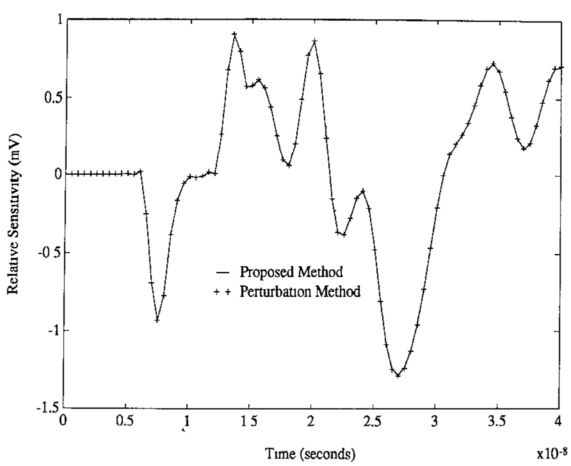

Fig. 13. Sensitivity of voltage V_{out} with respect to C_1 of Fig. 10.

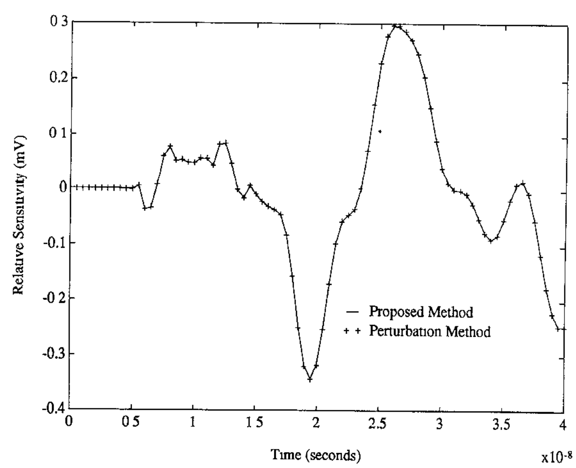

Fig. 16. Sensitivity of voltage V_{out} with respect to parameter $C_{1,2}$ of transmission line #1 in Fig. 10.

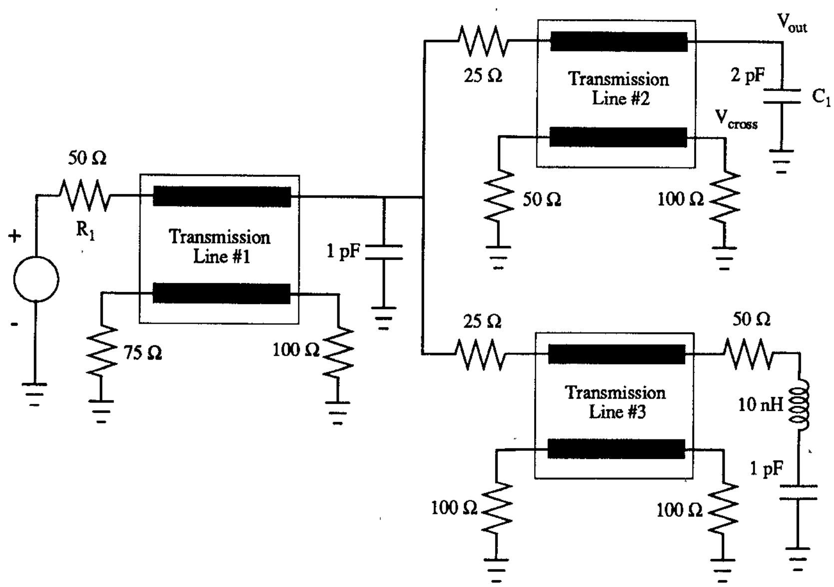

Fig. 17. Circuit for examples 3 and 4, interconnect model with lossy coupled transmission lines.

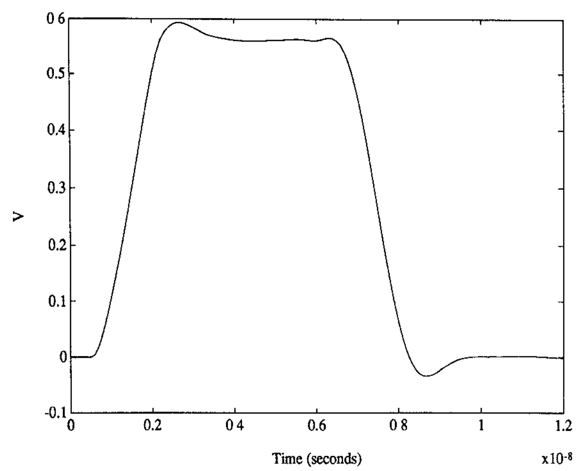

Fig. 18. Transient response of the circuit shown in Fig. 17 at node V_{out}.

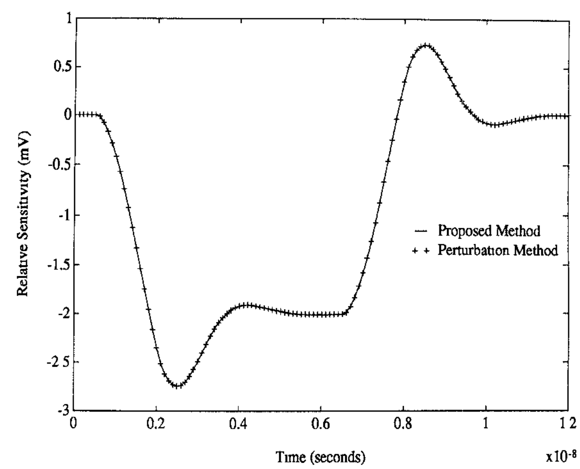

Fig. 20. Sensitivity of voltage V_{out} with respect to R_1 of Fig. 17.

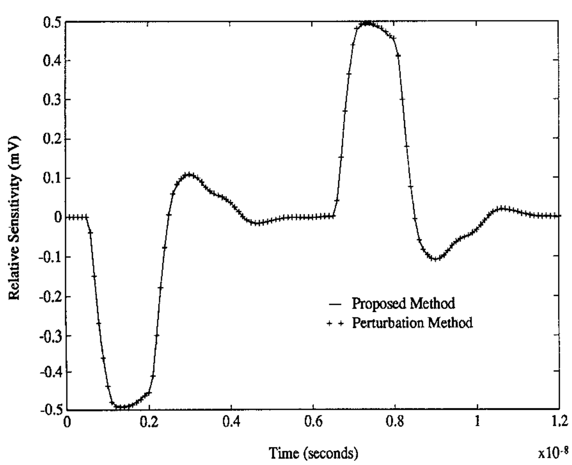

Fig. 19. Sensitivity of voltage V_{out} with respect to C_1 of Fig. 17.

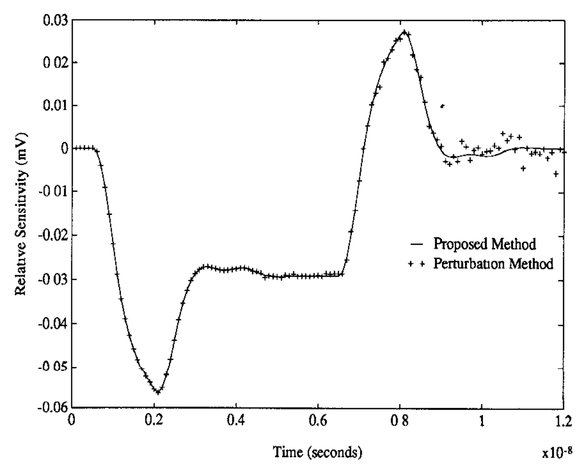

Fig. 21. Sensitivity of voltage V_{out} with respect to parameter $R_{1,1}$ of transmission line #2 in Fig. 17.

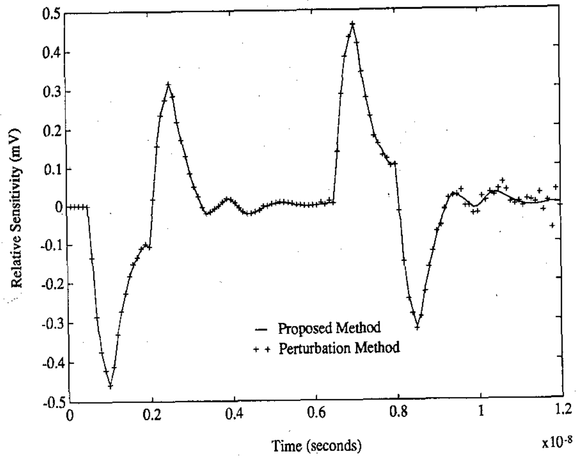

Fig. 22. Sensitivity of voltage V_{out} with respect to parameter $L_{1,1}$ of transmission line #2 in Fig. 17.

determined with the following equations:

$$C_{1,1}=C_{2,2}\approx \epsilon_r\epsilon_0 K_{C1}\left(\frac{w}{h}\right)+\frac{\epsilon_r\epsilon_0}{4\pi}K_{L1}K_{C1}\left(\frac{w}{h}\right)^2 \times \ln\left[1+\left(\frac{2h}{d}\right)^2\right]\ \text{F/m} \quad (34)$$

$$C_{1,2}=C_{2,1}\approx -\frac{\epsilon_r\epsilon_0}{4\pi}K_{L1}K_{C1}\left(\frac{w}{h}\right)^2 \times \ln\left[1+\left(\frac{2h}{d}\right)^2\right]\ \text{F/m} \quad (35)$$

$$L_{1,1}=L_{2,2}\approx \frac{\mu_r\mu_0}{K_{L1}}\left(\frac{h}{w}\right)-\frac{\mu_r\mu_0}{4\pi} \times \ln\left[1+\left(\frac{2h}{d}\right)^2\right]\ \text{H/m} \quad (36)$$

$$L_{1,2}=L_{2,1}\approx \frac{\mu_r\mu_0}{4\pi}\ln\left[1+\left(\frac{2h}{d}\right)^2\right]\ \text{H/m} \quad (37)$$

$$K_{L1}=\frac{120\pi}{Z_{0(\epsilon_r=1)}}\left(\frac{h}{w}\right) \quad (38)$$

$$K_{C1}=\left[\frac{120\pi}{Z_{0(\epsilon_r=1)}}\sqrt{\frac{\epsilon_{r(\text{eff})}}{K_{L1}\epsilon_r}}\left(\frac{h}{w}\right)\right]^2 \quad (39)$$

$$Z_0(\epsilon_{r=1})\approx 60\ln\left(\frac{8h}{w}+\frac{w}{4h}\right)\Omega \quad \text{for} \quad \frac{w}{h}\leqslant 1 \quad (40)$$

$$Z_{0(\epsilon_r=1)}\approx \frac{120\pi}{\frac{w}{h}+2.42-0.44\left(\frac{h}{w}\right)+\left(1-\frac{h}{w}\right)^6}\Omega \quad \text{for} \quad \frac{w}{h}\geqslant 1 \quad (41)$$

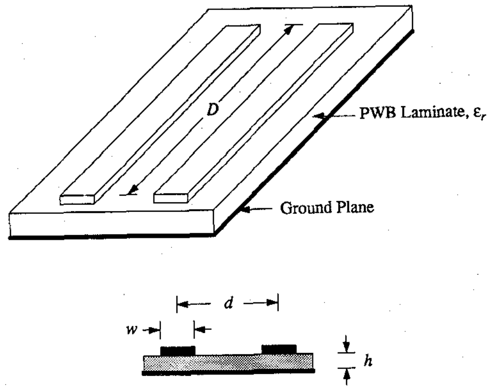

Fig. 23. Two conductor transmission line near a ground plane.

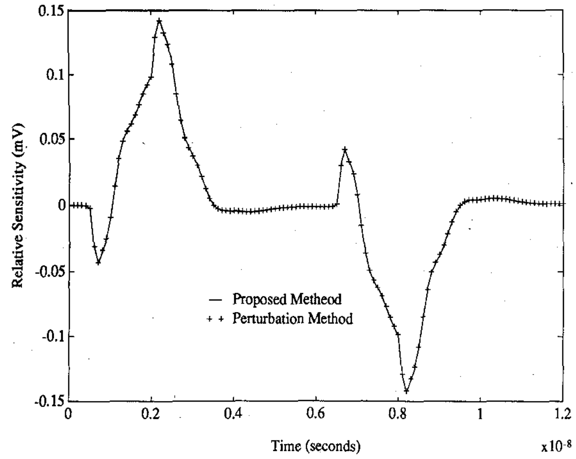

Fig. 24. Sensitivity of voltage V_{cross} with respect to the ratio w/d of transmission line #2 in Fig. 17.

where
ϵ_r is the relative dielectric constant,
$\epsilon_0 = 10^{-9}/36\pi$ F/m is the dielectric constant in vacuum,
ϵ_r(eff) is a function of h and w described in [18],
μ_r is the relative permeability and
$\mu_0 = 4\pi\times 10^{-7}$ H/m is the permeability in vacuum.

Using $w = 0.58$ mm, $h = 1.17$ mm and $d = 2.49$ mm, the following matrices of transmission line parameters are obtained:

$$\boldsymbol{L}=\begin{bmatrix}493.11 & 63.04\\ 63.04 & 493.11\end{bmatrix}\text{nH/m},$$

$$\boldsymbol{C}=\begin{bmatrix}69.62 & -7.09\\ -7.09 & 69.62\end{bmatrix}\text{pF/m}.$$

The circuit in Fig. 17 is used for this example. The transmission lines have the parameters listed above with the length of the lines being 0.05 m, 0.04 m and 0.03 m for transmission lines 1, 2, and 3, respectively. Fig. 24 is a plot of the sensitivity of V_{cross} to the ratio w/d of transmission line #2 keeping the ratio of h/w constant.

VII. Conclusion

An analysis method, based on the NILT algorithm, has been described for the evaluation of the time domain sensitivity of networks which contain lossy coupled transmission lines. The sensitivity can be calculated with respect to network components and electrical and physical parameters of the transmission lines. The efficiency of the proposed method makes it useful for determining critical network components and for generating sensitivities of output responses used in gradient based optimizers. A number of examples were presented showing the accuracy of the method compared to the perturbation technique.

References

[1] R. Sainati and T. Moravec, "Estimating high speed circuit interconnect performance," *IEEE Trans. Circuits Syst.*, vol. 36, pp. 533–541, Apr. 1989.

[2] H. Hasegawa and S. Seki, "Analysis of interconnection delay on very high-speed LSI/VLSI chips using a microstrip line model," *IEEE Trans. Electron Devices*, vol. ED-31, pp. 1954–1960, Dec. 1984.

[3] M. Nakhla, "Analysis of pulse propagation on high-speed VLSI chips," *IEEE J. Solid-State Circuits*, vol. 25, pp. 490–494, Apr. 1990.

[4] S. S. Gao, A. Y. Yang, and S. M. Kang, "Modelling and simulation of interconnection delays and crosstalks in high-speed integrated circuits," *IEEE Trans. Circuits Syst.*, vol. 37, pp. 1–9, Jan. 1990.

[5] D. Zhou, F. Preparata, and S. Kang, "Interconnection delay in very high-speed VLSI," in *Proc. IEEE Int. Conf. Computer-Aided Design* (ICCAD), Oct. 1988, pp. 52–56.

[6] A. R. Djordjević and T. K. Sarkar, "Analysis of time response of lossy multiconductor transmission line networks," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-35, pp. 898–908, Oct. 1987.

[7] M. Cases and D. Quinn, "Transient response of uniformly distributed RLC transmission lines," *IEEE Trans. Circuits Syst.*, vol. CAS-27, pp. 200–207, Mar. 1980.

[8] A. J. Groudis, "Transient analysis of uniform resistive transmission lines in a homogeneous medium," *IBM J. Res. Develop.*, vol. 23, pp. 675–681, Nov. 1979.

[9] A. J. Groudis and C. S. Chang, "Coupled lossy transmission line characterization and simulation," *IBM J. Res. Develop.*, vol. 25, pp. 25–41, Jan. 1981.

[10] R. Griffith and M. Nakhla, "Time-domain analysis of lossy coupled transmission lines," *IEEE Trans. Microwave Theory and Tech.*, vol. 38, pp. 1480–1487, Oct. 1990.

[11] K. Singhal and J. Vlach, "Computation of time domain response by numerical inversion of the Laplace transform," *J. Franklin Inst.*, vol. 299, no. 2, pp. 109–126, Feb. 1975.

[12] K. Singhal, J. Vlach, and M. Nakhla, "Absolutely stable, high order method for time domain solution of networks," *Archiv fur Electronik und Uebertragungstechnik*, vol. 30, pp. 157–166, 1976.

[13] C. W. Ho, A. E. Ruehli, and P. A. Brennan, "The modified nodal approach to network analysis," *IEEE Trans. Circuits Syst.*, vol. CAS-22, pp. 504–509, June 1975.

[14] S. W. Director and R. A. Rohrer, "Automated network design—the frequency domain case," *IEEE Trans. Circuit Theory*, vol. CT-16, pp. 330–337, Aug. 1969.

[15] A. E. Ruehli, "Inductance calculations in a complex integrated circuit environment," *IBM J. Res. Develop.*, vol. 16, pp. 470–481, Sept. 1972.

[16] C. Wei, R. F. Harrington, J. R. Mautz, and T. K. Sarkar, "Multiconductor transmission lines in multilayered dielectric media," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-32, pp. 439–450, Apr. 1984.

[17] J. Venkataraman, S. M. Rao, A. R. Djordjević, T. K. Sarkar, and Y. Naiheng, "Analysis of arbitrarily oriented microstrip transmission lines in arbitrarily shaped dielectric media over a finite ground plane," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, pp. 952–959, Oct. 1985.

[18] C. S. Walker, *Capacitance, Inductance and Crosstalk Analysis*. Norwood, MA: Artech House, 1990.

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